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The Complexity Course is a survey of techniques, applications, and implications of complexity science and complex systems. This course aims to be both an introduction for students from other fields, and a forum for continued discussion within the complexity community. Topics include systems dynamics, chaos, scaling, fat-tailed distributions, fractals, information theory, emergence, criticality, agent-based models, graph theory, and social networks.

Within each topic, we will focus on two complementary goals:

1. Building an intuition: Complexity is all around us, and complexity science is most useful when it can be easily applied to situations and research.
2. Modeling and analysis: Understanding complexity requires mathematical formalization and computational approaches, which are powerful and widely applicable.

Each week, we will have 4 hours of lecture. The first session each week will be an introduction to the contexts, intuition, and implications of a topic in complexity science, followed by student presentations and discussions of assigned readings. The second session will be a hands-on lab/lecture, where students learn to use computational techniques. Topics in the two lectures will build upon and support each other. Theory and mathematics will be used to delve deeper into the techniques and intuitions.

The goal of the paper discussions will be to collectively “reconstruct” the concepts developed in the papers: for each paper, one student presenter explains the models and what they teach us about the system under study, and another identifies potential problems and missed opportunities. Each participant will be responsible for playing one of these roles as a discussion leader two to three times throughout the semester. It is essential that everybody grapple with the readings ahead of the respective lecture session.

Applications will draw from climate science, ecology, conflict, historical physics, social theory, epidemiology, and governance.

The course will include 6 hours of tutorials in python, and tools for scientific work in python.
such as numpy and scipy. At the end of the semester, lab sessions will be partly used to help students workshop their work for their final projects.

The Course structure is summarized in the Table below, while the content of each session is detailed below.

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<th>Week</th>
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<th>Computation and Mathematics</th>
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In addition to readings and presentations, there are six lab assignments, and a final project. The lab assignments are as follows:

1. **Non-linearity and chaos**: characterizing the dynamics of iterative maps.
2. **Stochasticity**: fitting models to non-Gaussian data.
3. **Cellular automata**: constructing a random number generated from a cellular automaton.
4. **Self-organized criticality**: characterizing SOC noise and fractals.
5. **Agent-based models**: extending a human-environment spatial model with multiple agents.

Students choose their own final projects, either individually or as small groups. Each final project must apply techniques from the class to a context relevant to the student(s).

The grading breakdown is as follows:
- Labs: 50-60%
- Final Project: 25-35%
- Participation (including paper presentations): 15%

The 10% range for labs and projects is available for students who want to put more time into their final project, and who then would have about 1 lab less homework. PhD students who take this course for credit will be required to complete additional sections in the lab assignments.

**Schedule and Materials**

**Week 1: Introduction**
What is complexity? What is complexity science? What is it not? How have disciplines embraced complexity? Discussions of endogeneity and regimes.

**Readings:**

*Theory*
- Simple lessons from complexity by Goldenfeld, N. and Kadanoff, L., 1999
- Simplifying complexity: a review of complexity theory by Manson, S, 2000

*Supplementary:*
- Complexity, Economics, and Public Policy, by Durlauf, 2011
- ...Ant Fugue in Godel, Escher, Bach by Douglas R. Hofstadter

**Week 2: Systems Dynamics**
How does systems' dynamics represent dynamics? What can we learn from an SD model (tipping points, leverage, typical “shape” of time series). System models as ODEs.

Readings:

Theory

Applications

Tools
- Tests for building confidence in system dynamics models by Forrester, J.W. and Senge, P.M., 1978

Supplementary
- Sterman, J. 2002. All models are wrong: reflections on becoming a systems scientist.

Week 3: Non-linearity and Chaos
Implications of non-linearity in dynamical systems: chaos (sensitivity to initial conditions, impossibility of precise predictions, what we can learn from the system nevertheless, statistically and topologically), bifurcations (importance of understanding dependence on parameters), thresholds between regimes. Implications for management for resilience.

Readings:

Theory
- Chapter 12, “Strange Attractors”, In Non-linear Dynamics by S. Strogatz.

Applications

**Tools**

**Supplementary**
• Regime shifts, resilience, and biodiversity in ecosystem management by Folke, C., Carpenter, S., Walker, B., Scheffer, M., Elmqvist, T., Gunderson, L., Holling, C., 2004 (recommend to ecologists)
• Early Warning of unknown nonlinear shifts, by Carpenter, Brock, Ecology 2012.
• Poverty Traps and Appalachia, Durlauf 2011.

**Week 4: Stochasticity**
What are stochastic processes? What kinds of distributions do we see? Examples of fat-tail distributions and where they appear.
The effects of fat-tails: e.g., measures of biological diversity have no defined variance, the problem with climate prediction.
The consequences of non-ergodicity.
Techniques in modeling and estimating stochastic systems: monte-carlo, calibration on unstable surfaces (e.g., GA), markov chain, L1-norms, copulas, non-parametric models.

**Readings:**

**Theory**

**Applications**
**Tools**

- Think Stats, Allan Downey [exercises]

**Week 5: Information Theory**

Entropy, maximum entropy, information in Markov chains, and information flow through cellular automata grids. Computational complexity, and unknowability.

**Readings:**

**Theory**
- The Nature of Computation - Introductory chapter

**Application:**

**Supplementary**
- Some background on why people in the empirical sciences may want to better understand the information-theoretic methods by Anderson, 2003
  
  [http://aicanderson2.home.comcast.net/~aicanderson2/home.pdf](http://aicanderson2.home.comcast.net/~aicanderson2/home.pdf)

**Week 6: Scaling**

The nature of dimensionality, units, and dimensional analysis. Self-organized criticality, phase transitions, power-laws, and their implications.

**Readings:**

**Theory**
- Chapter 1 of Ubiquity: Why Catastrophes Happen by M. Buchanan
- Through the looking glass of complexity: the dynamics of organizations as adaptive and evolving systems, by Morel and Ramanujam, 2010
- Chapter 1 of Scaling by Barenblatt
- Power-law distributions in empirical data, by Clauset, Shalizi, and Newman, 2009

**Applications**
- Criticality and disturbance in spatial ecological systems, by Pascual and Guichard, 2005

Week 7: Fractals
Where do fractals appear in nature, and why? Topics in the mathematical theories surrounding fractals, and their usefulness for complex models.

Readings:

Theory
- Chapters 11 of Nonlinear Dynamics and Chaos by S. Strogatz: “Fractals”
- Chapter 8 of The Computational Beauty of Nature, by Flake, 1998

Applications
- River Networks as scale invariant phenomena: Section 3 of Dan Rothman’s Modeling Environmental Complexity Lecture Notes
- Chapter 3 of Fractals and Multifractals in Ecology and Aquatic Science, by Seuront, 2009
- Chapter 5 of Where medicine went wrong, by West, 2006

Week 8: Adding space and history
Consequences and metrics of spatial heterogeneity, proximity effects, local interactions to global effects, emergence. Path dependence, DNA and evolution and their computational analogs (genetic algorithms etc…). Turbulence and noise characteristics in space.

Readings

Theory
- The importance of being discrete (and spatial) by Durrett, R. and Levin, S., 1994
- Percolation: Section 6 of Dan Rothman’s Modeling Environmental Complexity Lecture Notes

Applications
- Agent-based modelling as scientific method: a case study analysing primate social behaviour by Bryson, Joanna J., Ando Yasushi and Hagen Lehmann
- Conquest and Regime Change: An Evolutionary Model of the Spread of Democracy and Peace by Cederman and Gleditsch.
Global Pattern Formation and Ethnic/Cultural Violence. May Lim et al. (2007)

Week 9: Graph Theory
Graphs as representations of relationships; common statistics on graphs and their implications; small-world and scale free graphs.

Readings:

Theory

Applications
- Introduction to Probabilities, Graphs and Causal Models, Chap 1, Causality, Judea Pearl

Tools
- Alan Downey, ThinkComplexity

Week 10: Social Networks
The applicability of networks to representing social relationships. Information flow through social networks. Central players and the significance of network statistics for social behaviors.

Readings:

Theory
- Network Analysis in the Social Sciences.
- Social Structure from Multiple Networks. I. Blockmodels of Roles and Positions by Harrison C. White, Scott A. Boorman and Ronald L. Breiger.
Applications


**Week 11: Dynamic Networks**


**Readings:**

*Theory*

- Robust Control and Hot Spots in Dynamic Spatially Interconnected Systems, by Brock and Xepapadeas

*Supplementary*


**Week 12: Cross-scales**

The problem of scales, hierarchy theory, and techniques for understanding cross-scale interactions and management. Scale separation versus self-similarity versus multiscale analysis.

**Readings:**

*Theory*

- The scale issue in social and natural sciences, by Danielle Marceau
- A hierarchical framework for the analysis of scale, O’Neill, Johnson and King.
- On time and space decomposition of complex structures, by P-J. Courtois
- Multiple scales and the maintenance of biodiversity, by Levin, 2000

*Applications*

- The Use of Discontinuities and Functional Groups to Assess Relative Resilience in
- The politics of scale, position, and place in the governance of water resources in the Mekong region, by Lebel, Garden, and Imamura, 2005

**Supplementary**
- Multiscale Low-frequency circulation modes in the global atmosphere, by Lau et al. 1994
- A Mathematical Theory of Strong Emergence Using Multiscale Variety, by Bar-Yam, Complexity 2004